

Exercise 5

Find the sum of the following infinite series:

$$e^{-2} + e^{-4} + e^{-6} + e^{-8} + \dots$$

Solution

Inspecting the series, we see that it is geometric. The first term is

$$a_1 = e^{-2},$$

and the common ratio is

$$r = e^{-2}.$$

Therefore, the sum of the series is

$$\begin{aligned} S &= \frac{a_1}{1 - r} \\ &= \frac{e^{-2}}{1 - e^{-2}} \\ &= \frac{1}{e^2 - 1}. \end{aligned}$$